

Solving the Graceful Exit Problem in Superstring Cosmology *

Sukanta Bose[†]

*Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind,
Pune 411007, India
(March 1997)*

Abstract

We briefly review the status of the “graceful exit” problem in superstring cosmology and present a possible resolution. It is shown that there exists a solution to this problem in two-dimensional dilaton gravity provided quantum corrections are incorporated. This is similar to the recently proposed solution of Rey. However, unlike in his case, in our one-loop corrected model the graceful exit problem is solved for any finite number of massless scalar matter fields present in the theory.

Pacs: 98.80.Cq, 04.60.Kz, 11.25.-w

Typeset using REVTeX

*Based on talk given at Conference on Big Bang and Alternative Cosmologies: A Critical Appraisal, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India, January 1997. To appear in the proceedings to be published by the Journal of Astrophysics and Astronomy (The Indian Academy of Sciences).

[†]Electronic address: *sbose@iucaa.ernet.in*

I. INTRODUCTION

The Standard Cosmological Model successfully explains many features related to the observed universe. However, it does not offer a solution to the initial singularity problem or account for the homogeneity and isotropy of the universe unless one invokes an ad hoc inflaton field and fine-tunes the initial conditions. Superstring cosmology appears to be promising in this regard. First, it is known to be well behaved at ultraviolet energy scales or at most have mild singularities (Jain 1997). Second, apart from the graviton, it has a naturally occurring scalar field, the dilaton, whose kinetic energy can be used to drive the universe through an inflationary phase (Veneziano, 1996). The solutions to the tree level effective action depict an FRW phase as well. Unfortunately, the tree level solution does not describe a smooth singularity-free transition from the inflationary phase to the FRW phase. This is called the “graceful exit” problem in superstring cosmology.

A quantum cosmology approach does indicate the possibility of a graceful exit (Gasperini et al 1996; Maharana et al 1997). More significantly, it was shown by S.-J. Rey (1996) that this problem is avoided in a string-inspired two-dimensional cosmological model provided back reaction effects to first order in the Planck constant are incorporated. However, Rey’s solution unphysically requires that the number of massless scalar fields N in the universe be less than twenty four! Some attempts, notably by Gasperini and Veneziano (1996), did not succeed in solving this problem.

In this paper, we begin by briefly describing the salient features of four-dimensional superstring cosmology in section II. We present a string-inspired classical action in two-dimensional dilaton gravity and illustrate the graceful exit problem in the context of its cosmological solutions in section III. In section IV we study the back-reaction effects due to one-loop quantum corrections on the spacetime geometry. With the addition of our choice of one-loop counterterms to the action we solve the graceful exit problem for any finite positive value of N . We conclude this paper with a discussion on the implications of our solution and possible scope for future research.

II. SUPERSTRING COSMOLOGY

The low energy limit of string theory is given by an effective action of the type

$$S_{\text{eff}} = \frac{1}{2} \int d^4x \sqrt{-g} e^{-2\phi} \left[\lambda_s^{-2} (\mathcal{R} + 4\partial_\mu \phi \partial^\mu \phi) - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + V \right] , \quad (2.1)$$

where ϕ is the dilaton field, \mathcal{R} is the four-dimensional Ricci scalar, $H_{\mu\nu\lambda}$ is the third-rank antisymmetric tensor field, and V is a term in which a dilaton potential or a cosmological constant term can be absorbed. There are two expansion parameters in the above action: $e^{2\phi}$ is the analogue of the Planck’s constant in quantum field theory and governs higher genus corrections. On the other hand λ_s^2 is related to the inverse of the string tension and controls string-size effects. This action resembles the Brans-Dicke action with $\omega = -1$. Just as in Brans-Dicke, here too calculations can be done in either the string (i.e., the Brans-Dicke) frame or the Einstein frame. The metric that appears in the above action is the string metric and in this paper we will base our discussion in this frame.

The above action has been shown to have cosmological solutions (for reviews see Veneziano (1996) and Gasperini (1996)). Typically a solution exhibits two branches defined by the range of the cosmic time τ . The branch corresponding to the range $-\infty < \tau \leq 0$ describes a superinflationary phase, where the scale factor grows as an inverse power-law in cosmic time. This phase is characterised by accelerated expansion and growing curvature. The branch with $0 \leq \tau < \infty$ describes an FRW phase.

A crucial problem facing string cosmology is the lack of a smooth transition from the superinflationary phase to the FRW phase. This is because the superinflationary phase ends up in a region of diverging scalar curvature and coupling. This is the graceful exit problem in the context of superstring cosmology. There are no-go theorems (Brustein & Veneziano 1994; Kaloper et al 1995) that show that even in the presence of realistic dilaton potentials a graceful exit from accelerated inflation does not occur, without invoking corrections from string-size effects (see, however, Kalyana Rama 1997). In the subsequent sections we address this issue in cosmological models of string-inspired two-dimensional dilaton gravity.

III. CLASSICAL TWO-DIMENSIONAL COSMOLOGY

A classical two-dimensional (2D) theory that describes cosmological models of interest is given by the action of Callan et al (1992):

$$S_0 = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left\{ e^{-2\phi} [R + 4(\nabla\phi)^2 - 4\Lambda] - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right\} , \quad (3.1)$$

where Λ is a cosmological constant term, and f_i 's are N massless scalar matter fields. In particular, the above action has been shown to yield cosmological solutions with a superinflationary branch disconnected from an FRW branch (Rey 1996; also see Mazzitelli & Russo 1993).

Varying the above action with respect to the metric, dilaton, and the scalar fields gives the following equations of motion:

$$\begin{aligned} 2e^{-2\phi} \left[\nabla_\mu \nabla_\nu \phi + g_{\mu\nu} ((\nabla\phi)^2 - \nabla^2\phi + \Lambda) \right] + \frac{1}{4} g_{\mu\nu} \sum_{i=1}^N (\nabla f_i)^2 - \frac{1}{2} \sum_{i=1}^N \nabla_\mu f_i \nabla_\nu f_i &= 0 , \\ e^{-2\phi} \left[R - 4\Lambda + 4\nabla^2\phi - 4(\nabla\phi)^2 \right] &= 0 , \quad \text{and} \quad \nabla^2 f_i = 0 . \end{aligned} \quad (3.2)$$

In the conformal gauge, $g_{\mu\nu} \equiv e^{2\rho} \eta_{\mu\nu}$, the metric components in the double null-coordinates, $x^\pm = t \pm x$, are $g_{+-} = -\frac{1}{2}e^{2\rho}$ and $g_{++} = g_{--} = 0$. In this gauge, the two-dimensional Ricci scalar is $R = 8e^{-2\rho} \partial_+ \partial_- \rho$, and the equations of motion take the form

$$\phi : \quad e^{-2(\phi+\rho)} \left[-4\partial_+ \partial_- \phi + 4\partial_+ \phi \partial_- \phi + 2\partial_+ \partial_- \rho - \Lambda e^{2\rho} \right] = 0 , \quad (3.3)$$

$$\rho : \quad e^{-2\phi} \left[2\partial_+ \partial_- \phi - 4\partial_+ \phi \partial_- \phi + \Lambda e^{2\rho} \right] = 0 . \quad (3.4)$$

Since we have gauge fixed g_{++} and g_{--} to zero we must also impose their equations of motion as constraints. This gives

$$e^{-2\phi}(4\partial_{\pm}\rho\partial_{\pm}\phi - 2\partial_{\pm}^2\phi) = -\frac{1}{2}\sum_{i=1}^N\partial_{\pm}f_i\partial_{\pm}f_i. \quad (3.5)$$

Adding Eqs. (3.3) and (3.4) gives the continuity equation $\partial_+\partial_-(\rho - \phi) = 0$, which shows that $(\rho - \phi)$ behaves as a free field.

In this paper we will discuss only the case of a vanishing cosmological term ($\Lambda = 0$). The homogeneous cosmological solutions are best described in terms of the new fields $\Phi \equiv e^{-2\phi}$ and $\Sigma \equiv (\phi - \rho)$. In vacuum, i.e., for $\dot{f}_i = 0$ (where an overdot denotes $\partial/\partial t$), for all values of i , the equations of motion (3.3) simplify to $\ddot{\Phi} = \ddot{\Sigma} = 0$. The corresponding solution is

$$-\Sigma \equiv (\rho - \phi) = Q_{\Sigma}t + A, \quad \Phi \equiv e^{-2\phi} = Q_{\Phi}t + B, \quad (3.6)$$

where Q_{Σ} , Q_{Φ} , A , and B are integration constants determined by initial conditions. On the other hand, the classical constraints (3.5) yield

$$\ddot{\Phi} + 2\dot{\Phi}\dot{\Sigma} = -Q_{\Phi}Q_{\Sigma} = 0, \quad (3.7)$$

which shows that the cosmological solutions have two distinct branches depending on whether Q_{Σ} or Q_{Φ} is non-zero.

The solution in the first branch ($Q_{\Phi} \neq 0, Q_{\Sigma} = 0$) is given by

$$\rho = \phi + \ln 2C; \quad e^{-2\phi} = -\frac{8C^2t}{M}, \quad (3.8)$$

where C and M are constants. For real values of the coupling e^{ϕ} this solution describes the universe for only $t < 0$. From above, the spacetime metric for $t < 0$ is

$$(ds)^2 = -\left(\frac{M}{-2t}\right)[dt^2 - dx^2] = -[d\tau^2 - \left(\frac{M}{-\tau}\right)^2 dx^2], \quad (3.9)$$

where $\tau \equiv -(-2Mt)^{1/2}$ is the comoving time. In the comoving coordinates the scale factor is $a(\tau) = M/(-\tau)$, which depicts a superinflationary evolution.

The solution in the second branch ($Q_{\Sigma} \neq 0, Q_{\Phi} = 0$) is given by

$$\rho = \phi + \tilde{M}t; \quad e^{-2\phi} = \tilde{C}^{-2}, \quad (3.10)$$

which describes an “expanding” universe with the metric

$$(ds)^2 = -\tilde{C}^2 e^{2\tilde{M}t}[dt^2 - dx^2] = -[d\tau^2 - (\tilde{M}\tau)^2 dx^2], \quad (3.11)$$

for non-negative values of the comoving time, $\tau \equiv (\tilde{C}/\tilde{M}) \exp \tilde{M}t$. The dilaton, and therefore the coupling, is constant in this branch of the universe.

The two-dimensional universe described by Eqs. (3.9) and (3.11) have two notable features. First, as in higher dimensions, the two branches are related by scale factor duality (Veneziano 1991; Meissner and Veneziano 1991; Sen 1991; Sen 1992; Hassan & Sen 1991). Second, the superinflationary branch described by Eq. (3.9) terminates in a region of diverging scalar curvature and coupling as $\tau \rightarrow 0^-$. Thus, as in higher dimensions, a smooth transition from the superinflationary branch to the FRW phase does not occur in this classical theory.

IV. INCORPORATING ONE-LOOP CORRECTIONS

In the superinflationary branch (3.9), well before $\tau \rightarrow 0^-$, the universe enters a region of strong coupling where corrections due to quantum gravitational effects become non-negligible. In this regime the predictions of the classical theory cannot be trusted and higher order corrections to the metric and the dilaton must be incorporated. Such an attempt was made by Mazzitelli & Russo (1993) and Rey (1996) by including one-loop corrections to the classical action. The particular one-loop corrected model they considered is the RST model (see Russo et al 1992). Mazzitelli & Russo showed that in the one-loop corrected model a smooth transition from the superinflationary phase to the FRW phase is not possible for negative values of Λ . However, Rey showed that the graceful exit problem is solved in this model for $\Lambda = 0$ *provided* the number of massless scalar fields N is less than 24.

In this paper we propose the following one-loop corrected model, which is different from RST, and study its cosmological solutions:

$$S_1 = S_0 + \frac{N\hbar}{24\pi} \int d^2x \sqrt{-g} \left(-\frac{1}{4} R \square^{-1} R + 2(\nabla\phi)^2 - 3\phi R \right) , \quad (4.1)$$

where $\square_x G(x, x') = \delta^2(x - x') / \sqrt{-g(x)}$. The first term in the parenthesis is the Polyakov-Liouville term that reproduces the trace anomaly for massless scalar fields (Callan et al 1992). However the one-loop action is defined only up to the addition of local covariant counterterms (Russo & Tseytlin 1992). Our action differs from RST only in the addition of different counterterms. The higher order corrections beyond one loop are dropped by using the large N approximation where $N \rightarrow \infty$ as $\hbar \rightarrow 0$ such that $\kappa \equiv N\hbar/12$ remains finite.

We now use the following one-loop corrected redefined fields

$$\Sigma \equiv (\phi - \rho), \quad \Phi \equiv e^{-2\phi} - \kappa\phi + \frac{\kappa}{2}\rho = e^{-2\phi} - \frac{\kappa}{2}\rho - \kappa\Sigma . \quad (4.2)$$

For homogeneous cosmologies with constant f_i 's, the equations of motion in terms of these variables take the same form as in the classical case, i.e., $\ddot{\Phi} = 0 = \ddot{\Sigma}$. However the constraints get modified to

$$\partial_{\pm}^2 \Phi + 2\partial_{\pm} \Phi \partial_{\pm} \Sigma = \frac{3}{2}\kappa \left[\partial_{\pm}^2 \phi - 2\partial_{\pm} \rho \partial_{\pm} \phi \right] + \kappa t_{\pm}(x^{\pm}) , \quad (4.3)$$

where $t_{\pm}(x^{\pm})$ are nonlocal functions that arise from the homogeneous part of the Green function (see Callan et al 1992; Bose et al 1995). The choice of these nonlocal functions determine the quantum state of the matter fields in the spacetime. The total matter stress tensor can be expanded in orders of \hbar as $T_{\mu\nu}^f = (T_{\mu\nu}^f)_{cl} + \langle T_{\mu\nu} \rangle$, where $(T_{\pm\pm}^f)_{cl} \equiv \frac{1}{2} \sum_{i=1}^N (\partial_{\pm} f_i)^2$ and $\langle T_{\pm\pm} \rangle = \kappa [\partial_{\pm}^2 \rho - (\partial_{\pm} \rho)^2 - t_{\pm}(x^{\pm})]$ is the one-loop contribution (Davies et al 1976). We will choose the state of the matter fields to be defined by

$$t_{\pm}(x^{\pm}) = -\frac{3}{2} \left[\partial_{\pm}^2 \phi - 2\partial_{\pm} \rho \partial_{\pm} \phi \right] . \quad (4.4)$$

The equations of motion, $\ddot{\Phi} = 0 = \ddot{\Sigma}$, yield the following solution

$$-\Sigma \equiv (\rho - \phi) = Q_\Sigma t + A, \quad \Phi \equiv e^{-2\phi} - \frac{\kappa}{2}\rho - \kappa\Sigma = Q_\Phi t + B. \quad (4.5)$$

Whereas the constraint (4.3), under the condition (4.4), yields the same classical expression (3.7). Once again depending on whether Q_Φ or Q_Σ is non-zero, one finds two branches of the solution. However, unlike the classical case, in this one-loop corrected model each branch separately describes smooth transition from a superinflationary phase to an FRW phase.

The first branch is given by $\rho = \phi + \ln 2C$ and $e^{-2\phi} - \kappa\rho/2 = -8C^2t/M$, where, unlike in Rey (1996), κ is now positive. This solution can be reexpressed as

$$e^{-2\rho} - \frac{\kappa}{8C^2}\rho = -\frac{2t}{M} \quad \text{and} \quad e^{-2\phi} - \frac{\kappa}{2}\phi = -\frac{8C^2t}{M} + \frac{\kappa}{2}\ln 2C. \quad (4.6)$$

This solution is valid for all real values of the conformal time t and for $\kappa > 0$. At asymptotic past timelike infinity, $t \rightarrow -\infty$, the metric and the dilaton approach the forms (Rey 1996)

$$(ds)^2 \rightarrow -\left(\frac{M}{-2t}\right)[dt^2 - dx^2] = -[d\tau^2 - \left(\frac{M}{-\tau}\right)^2 dx^2], \quad \text{and} \quad \phi \rightarrow -\ln(-2\tau), \quad (4.7)$$

where $\tau \equiv -(-2Mt)^{1/2}$. As $t \rightarrow \infty$

$$(ds)^2 \rightarrow -e^{32C^2t/\kappa M}[dt^2 - dx^2] = -[d\tau^2 - \left(\frac{16C^2}{\kappa M}\tau\right)^2 dx^2] \quad \text{and} \quad \phi \rightarrow \ln \tau, \quad (4.8)$$

where the comoving time is $\tau \approx (\kappa M/16C^2)\exp(16C^2t/\kappa M)$. Thus in the one-loop corrected model the universe begins in a classical superinflationary phase and ends up being in an FRW phase. The solution corresponding to the second branch is different from Rey's and is discussed elsewhere (Bose & Kar 1997).

The remaining question we would like to address is whether our one-loop corrected model indeed displays a smooth transition between the two phases in each branch. This can be checked by verifying that the scalar curvature remains finite at all times and the coupling remains small always, such that the large N approximation is not violated. These requirements can be shown to hold for the one-loop corrected solution (4.6) (on the same lines as Rey (1996), but now for any finite positive value of N). A detailed discussion of the exact solutions to our model (4.1) is given in Bose & Kar (1997). There we also address the question of how scale factor duality affects graceful exit.

V. DISCUSSION

Above we proposed a two-dimensional model in dilaton gravity that solves the graceful exit problem for any finite positive value of N . As shown by Eq. (4.4) this solution requires the presence of a homogeneous distribution of massless scalar matter fields. Further, it can be shown that the Weak Energy Condition (WEC) is violated in this solution (Bose & Kar 1997). In fact this agrees with the recent result of Brustein & Madden (1997) and Kar (1996).

The next logical step is to account for string-size corrections in the 4D cosmological models. One expects that this might solve the graceful exit problem in 4D. However, a

perturbative solution, as the one given here for 2D, is unlikely to do the job. A more fruitful approach, as advocated by Brustein and Veneziano (1994), might be to look for a conformal field theory (possessing cosmological solutions) endowed with appropriate duality symmetries that can be exploited to probe the strong coupling regions successfully.

I am grateful to J. V. Narlikar and G. Veneziano for motivating me to look into this problem. I thank S. Kalyana Rama, J. Maharana, T. Padmanabhan, S. Panda, V. Sahni, and especially S. Kar and S. Sinha for helpful discussions.

REFERENCES

- Bose, S., Kar, S. 1997, *in preparation*.
- Bose, S., Parker, L., Peleg, Y. 1995, *Phys. Rev.*, **D52**, 3512.
- Brustein, R., Madden, R. 1997, Ben-Gurion University Report No. BGU-PH-97/06 (hep-th/9702043).
- Brustein, R., Veneziano, G. 1994, *Phys. Lett.*, **B329**, 429.
- Callan, C.G., Giddings, S., Harvey, J., Strominger, A. 1992, *Phys. Rev.*, **D45**, 1005.
- Davies, P. C. W., Fulling, S. A., Unruh, W. G. 1976, *Phys. Rev.*, **D13**, 2720.
- Gasperini, M. 1996, in *String Gravity and Physics at the Planck Energy Scale*, eds. N. Sanchez and A. Zichichi (Kluwer Academic Publishers, Netherlands), p. 305.
- Gasperini, M., Maharana, J., Veneziano, G. 1996, Report Nos. CERN-TH/96-32 and IP-BBSR/96-9 (hep-th/9602087).
- Gasperini, M., Veneziano, G. 1996, *Phys. Lett.*, **B387** 715.
- Hassan, S.F., Sen, A. 1991, *Nucl. Phys.*, **B375**, 103.
- Jain, S. 1997, see the article in this volume.
- Kaloper, N., Madden, R., Olive, K. A. 1995, *Nucl. Phys.* **B452**, 677.
- Kalyana Rama, S. 1997, *Phys. Rev. Lett.*, **78**, 1620.
- Kar, S. 1996, in *Scale Factor Duality and the Energy Condition Inequalities*, Report No. IUCAA-43/96 (hep-th/9611122).
- Maharana, J., Mukherji, S., Panda, S. 1997, in *Notes on Axion, Inflation and Graceful Exit in Stringy Cosmology*, Report Nos. IP-BBSR/96-67, CTP-TAMU-63/96, and MRI-PHYS/96/34 (hep-th/9701115).
- Mazzitelli, F. D., Russo, J. G. 1993, *Phys. Rev.*, **D47**, 4490.
- Meissner, K. A., Veneziano, G. 1991, *Phys. Lett.*, **B267**, 33.
- Rey, S.-J. 1996, *Phys. Rev. Lett.*, **77**, 1929.
- Russo, J. G., Susskind, L., Thorlacius, L. 1992, *Phys. Rev.*, **D46**, 3444.
- Russo, J. G., Tseytlin, A. 1992, *Nucl. Phys.*, **B382**, 259.
- Sen, A. 1991, *Phys. Lett.*, **B271**, 295.
- Sen, A. 1992, *Phys. Lett.*, **B274**, 34.
- Veneziano, G. 1991, *Phys. Lett.*, **B265**, 287.
- Veneziano, G. 1996, in *String Gravity and Physics at the Planck Energy Scale*, eds. N. Sanchez and A. Zichichi (Kluwer Academic Publishers, Netherlands), p. 285.